DETERMINATION OF NEWTON'S GRAVITATIONAL CONSTANT, G, WITH IMPROVED PRECISION

Status Report
covering the period
1 October 1965 - 31 March 1966
under NASA Grant NGR-47-005-022

Principal Investigators
J. W. Beams
Department of Physics

A. R. Kuhlthau
R. A. Lowry
H. M. Parker
Department of Aerospace Engineering
and Engineering Physics

Research Laboratories for the Engineering Sciences

University of Virginia

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Report No. EP-4028-103-66U
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RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
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SECTION I
INTRODUCTION

This is the third semi-annual status report under this grant, the object of which is to apply new techniques to the laboratory determination of the Newtonian Gravitation Constant, G, with the expectation of improvement of the accuracy by about two orders of magnitude. The improvement is attributed to both a new approach to the measurement, which should eliminate or bring under better control most of the difficulties encountered in earlier methods, and to improvements in materials, metrology, fabrication procedures, etc. which should circumvent many problems and limitations previously encountered.

With reference to the schematic diagram of Figure 1, two small masses, m, are connected by a rigid horizontal rod. These are then placed in proximity to another pair of masses, M, and the gravitational interaction between the two sets of masses results in a pure torque acting on the m-system. A measure of this torque plus a knowledge of the spearation distance, d, enables G to be determined. Previous experimenters either measured the torque directly by observing the static deflection of a torsion fiber suspension or introduced variations on this method such as inducing resonant torsional oscillations in the m-system by suitably driving the large M system.

In the present proposed method the gravitational torque is maintained constant by detecting changes in the relative angular position β of the two mass systems and correcting the relative position by a servo mechanism to maintain β constant to the necessary accuracy. Thus the nearly constant torque being applied to the m-system results in a nearly constant angular acceleration of this system which can be permitted to act for an extended period of time. The ultimate measurement, then, is one of determining the angular speed of the m-system at the end of this extended period of time, i.e., of measuring large displacements and long times, both of which can be done accurately.

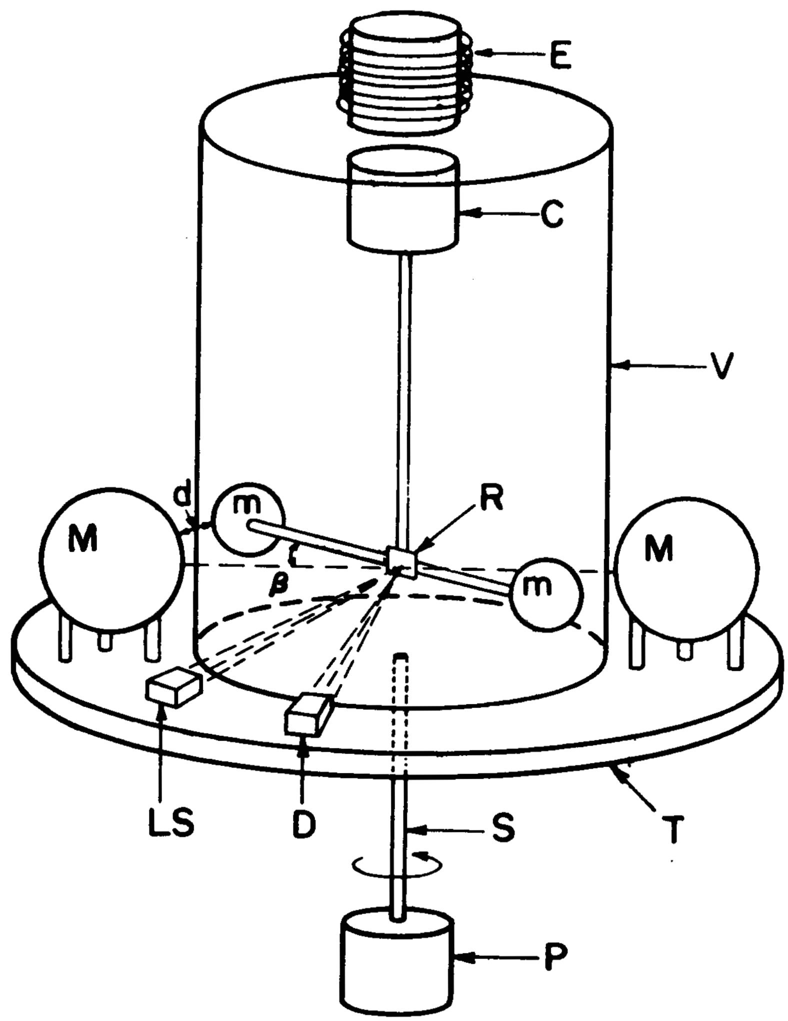


FIGURE 1

SCHEMATIC OF APPARATUS

In designing the experiment, three methods for suspending the m-system have been considered. Two of these involve the use of magnetic suspensions and for future reference these will be designated Models A-1 and A-2. The other involves the use of a more conventional torsion fiber and this will be designated as Model B. Each system has many advantages and disadvantages in comparison with the others, and in fact it is at this stage impossible to make the final selection. They are all being studied more carefully as part of the initial phases of the program, and working models of each design are being constructed and evaluated.

For the orientation of the reader, the principal differences are:

Model A-1 and A-2 - Magnetic Support

Again referring to Figure 1, the center of the rod connecting the small masses, m, is attached to a rigid vertical rod containing at its upper end, a small cylinder of magnetic material, C. This is freely suspended by the electromagnet E [1]. The major advantage of the electromagnetic suspension models is that the precision to which the angle β must be held constant in order to get the desired precision in the determination of G is only 10^{-3} radians, an easy prospect. The major disadvantage is the rather elaborate magnetic shielding required.

In Model A-1 the coil of the electromagnet E, is attached to the rotating table.

In Model A-2 the coil of the electromagnet is free of the rotating table and is fixed in the laboratory frame of reference.

The advantages and disadvantages between these two versions are more subtle. They are discussed in more detail in reference [2], and for the present purposes it need only be said that they do exist, are significant, and can only be evaluated by further experimentation.

Model B - Torsion Fiber

Model B replaces the electromagnetic suspension with a delicate torsion fiber mounted from the rotating table. As the m-system begins to rotate as a result of its interaction with the M system, the fiber support is rotated, due to the table rotation, and the balance always operates at essentially constant deflection (which will be as close as possible to the null point). The major disadvantage here is that this deflection angle and β, must be held constant to 10^{-6} radians in order to get accuracy in the G measurement comparable to what can be obtained by knowing β to 10^{-3} radians in the case of the magnetic support. The big advantage, of course, is the elimination of the magnetic field, and hence a minimization of magnetic effects.

At the present time the program is divided into three principle tasks, the status of each which is summarized in the following sections. These tasks are:

1. Theoretical Analysis
2. Apparatus Design - Details and Procurement of Components
3. Assembly of Apparatus and Development of Techniques.

SECTION II
CURRENT STATUS OF THE PROGRAM

A. Task I - Theoretical Analysis

The theoretical studies can be grouped under several major categories.

1. Geometry and Density Variations

The effects of geometry variations are considered to be reasonably well in hand. Such analyses do not, of course, solve the experimental problem of achieving good geometry. However, they should serve as a basis for "error experiments" to permit the evaluation and reduction of geometry errors.

The emphasis has now shifted to a detailed study of the effect of density variations in the interacting masses (especially in the large masses where they might be expected to be significant). The present approach is to develop a fundamental analysis of the problem starting with a suitable expansion for the gravitational potential of a large mass. While such an analysis should be possible (even though probably quite tedious) the translation of the results into a practically feasible set of "error experiments" to permit the evaluation and reduction of density inhomogeneity will require some serious study.

2. Equation of Motion

Since the last status report (4) two reasonable significant developments in the motion analysis have been made.

(a) The effect of masses fixed in the laboratory.

The basic motion equation of the systems is derived by writing an expression for all the torques acting on the m-system due to those masses fixed on the rotating table and those fixed in the laboratory system. The expression for the latter group of torques is then expanded in powers of $w = \frac{r_1}{d_i}$, where r_1 is the half length of the small mass system and d_i is the distance from the center of the small mass system to the center of i^{th} fixed laboratory mass, μ_i. Retaining only the first term in the above expansion the following motion equation is obtained.

$$(1) \qquad \ddot{\theta} = a + \sum_i A_i \sin(2\theta + \phi_i)$$

where:

a = constant angular acceleration due to masses fixed on the rotating table.

$\sum_i A_i \sin(2\theta + \phi_i)$ - first approximation to the acceleration due to i masses fixed in the laboratory.

It has been shown that equation (1) can be transformed to

$$(2) \qquad \ddot{\theta} = a + A_{eff} \sin(2\theta + \theta_{eff}).$$

Thus an arbitrary distribution of mass fixed in the laboratory system has the same effect on the motion as an effective single mass. Moreover, it has been shown to be possible to fix an additional suitable single mass in the laboratory so that the resulting A_{eff} is zero. Since A is proportional to $\frac{u}{d^3}$ considerable leeway in choosing the mass or d is possible.

If higher order terms in u are retained in the torque expression (the second term αu^3 gives rise to additional sin (2θ) terms as well as sin (4θ) terms in equation (1)), the reduction of sums to a single effective term can still be made. The same form of the solution to the motion equation holds, but it has not yet been established that cancellation can be effective with a single additional mass.

(b) Design of experiments.

Based on the results of (a) above, several experiments have been designed to test the effect of masses fixed in the laboratory. A comparison of calculated and experimentall observed effects on the motion of a laboratory mass should provide valuable information concerning the experimental precision being obtained, validity of the solution of the motion equation, etc. Such experiments will soon be performed with the prototype apparatus, and later with the precision apparatus.

3. <u>Data Analysis</u>

The solution of the motion equation has been shown to be

$$(3) \qquad t_n = \frac{\dot{\theta}_0}{a} \left\{ \sqrt{1 + \frac{4\ na}{\dot{\theta}_0^{\,2}}} - 1 \right\} + K_1 n + k_2 n^2 + \ldots k_i n^i + \ldots$$

where

(4) $a = \frac{4}{3} \pi \; MGX$

M = mass of a large mass

X = geometric factor depending on the type of small mass system used.

Thus the ultimate measurement of G will come from these equations. A computer program to make a least squares fit of the data to equation (3) is virtually completed. Provision to use any number of the constants, K, between zero and four has been built into the program.

4. Design of the Mass Systems

The decisions as to the size of the overall mass systems and the shape of the small mass system have not yet been made. The factor X of equation (4) for a cylindrical rod has not yet been calculated. The problem has been formulated but a solution in closed form does not seem imminent. Thus a computer program is being designed to determine the result to the desired degree of precision.

5. Error Analysis

Some effort has been devoted to the design of experiments to demonstrate the accuracy of the final measurement. It is, of course, apparent that in an experiment in which the primary purpose is to achieve the precision measurement of some quantity, the bulk of the actual experimental work is likely to be concerned with determining the accuracy of the final result. This process of analyzing, designing experiments, analysis of data, and ferreting out systematic errors is expected to be a continuing and important one. A study of the obvious effects is easy. The problem is to find the not-so-obvious ones, and even to determine whether or not they exist.

B. Task II - Apparatus Design and Procurement of Components

The following information supplements that already given in previous reports concerning the various subsystems of the apparatus.

1. Large Mass System

The decision has been made to use specially prepared tungsten spheres of a nominal 4 inch diameter and a purchase order was placed in September

1965 for their fabrication by the Y-12 plant of the USAEC, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee. Work is proceeding more slowly than had been anticipated with the major delay being due to difficulties encountered by the fabricator in obtaining the proper tungsten material from their supplier. Fortunately, much preliminary work remains to be done in the overall program before it will be necessary to have the precision large masses. Hence this delay in their fabrication should cause little or no delay in the final measurements.

2. Small Mass System

A decision on the final design of the small mass system is being postponed as long as possible. It is expected to be somewhat easier to fabricate than the large masses and, similarly, it is not required until later in the program. Tentative plans still call for the use of a rod suspended about an axis perpendicular to its own axis, with the rod most probably being made from quartz.

3. Rotary Table

As mentioned in the previous report the precision spindle upon which the rotary table is mounted had considerably more rotating bearing friction than had been originally anticipated. After some study of this problem, it was decided that this was undoubtedly unavoidable if extreme precision was required in the spindle. Thus the other parts of the system affected by this factor would have to be modified accordingly.

4. Servo Drive System

This was described in some detail in the previous report (4). The large AC torque motor has been installed and it may be satisfactory. It has received extensive testing and it appears to develop sufficient power and is able to be controlled smoothly at the very slow rotation rates desired for the experiment. The one remaining problem is the rather large heat dissipation in the motor. It runs quite hot and it has been necessary to install extra cooling fins on the stator. These seem to provide satisfactory motor performance but the heat source in the vicinity of the precision apparatus may cause difficulty in achieving thermal stability in the apparatus. However, it is encouraging that heating of the plane of the rotating table seems to be small. Thus the combination of large radiating surface and large heat capacity in the spindle, table, etc. may minimize the problem.

5. <u>Measurement of Period of Rotation of Rotary Table</u>.

A general description of the method chosen to make this measurement was presented in the first progress report (3). However, the engineering details were not pursued in depth until the present period, and a description of the first approach follows.

Basically, a theoretical study of the overall experiment clearly demonstrates that the ultimate in precision will be obtained by securing as much information as possible during an experiment which will run over an extended period of time. Thus, it is considered highly desirable to determine each revoltuion to an accuracy of 10^{-5} radians. Commercially available counting equipment is quite adequate for this purpose, but the problem lies in relating the triggering pulse, which must be supplied to the counters, to a position on the table. A rotational speed averaging about 1 revolution per minute appears to be an optimum operational speed. Running at this speed the table will rotate through an angle of 10^{-5} radians in 68 microseconds. During this time a point on the periphery of the table moves only 60 micro inches. The main problem, then, is to generate a suitable triggering signal while a reference point lies within this 60 micro inch interval.

The method which has been developed for generating these signals and activating the counters is shown in Figure 2.

Light from the lamp on the right hand side of the diagram is concentrated and directed through an aperature .01 inches in diameter. From this first aperature the beam diverges and fills the 6 inch parabolic mirror which is rigidly mounted to the rotating table. When the table is properly aligned the light converges and passes through a second .01 inch aperature near the first one. After passing through the second aperature the beam again diverges and strikes a photodiode generating a voltage pulse.

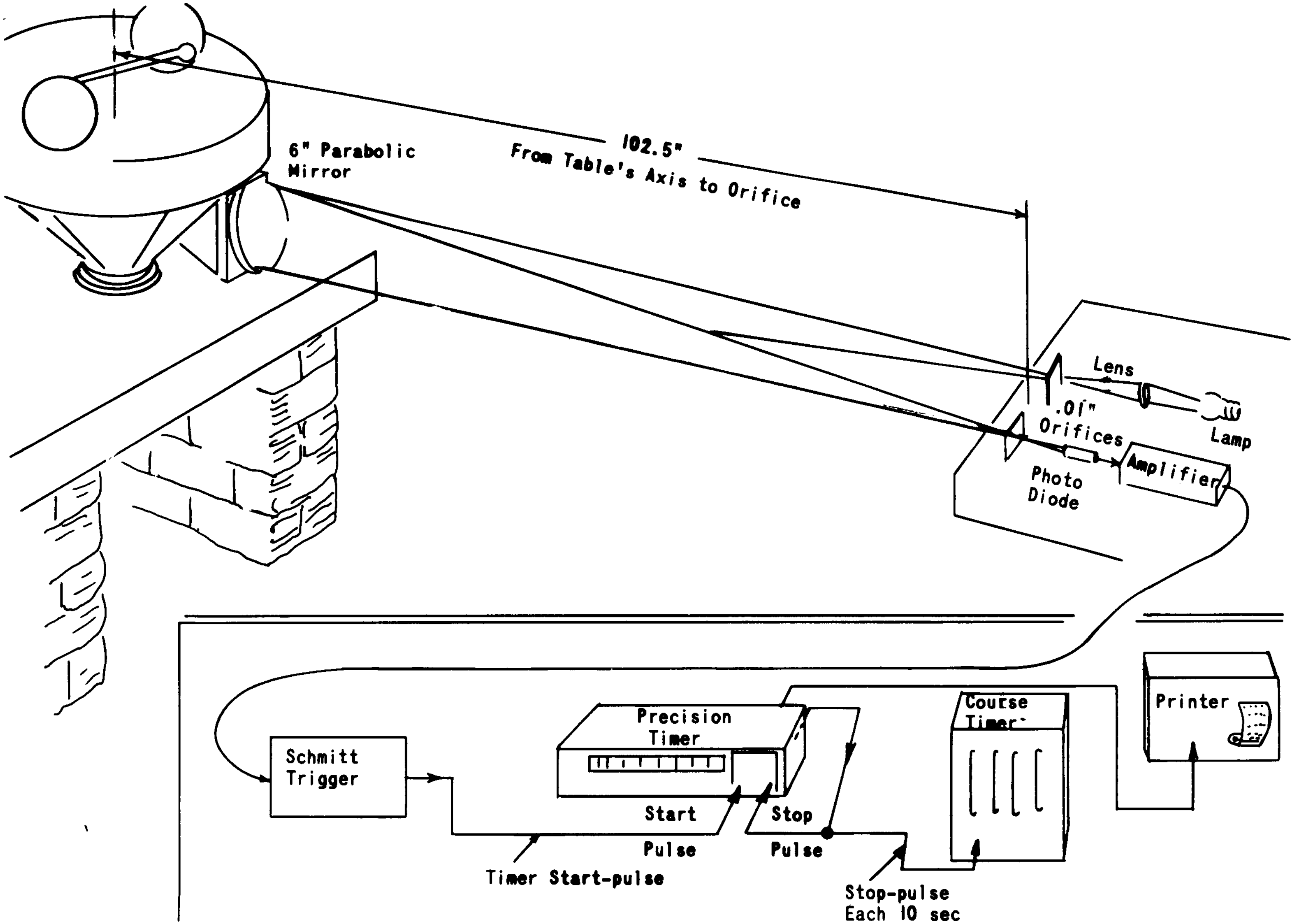


FIGURE 2

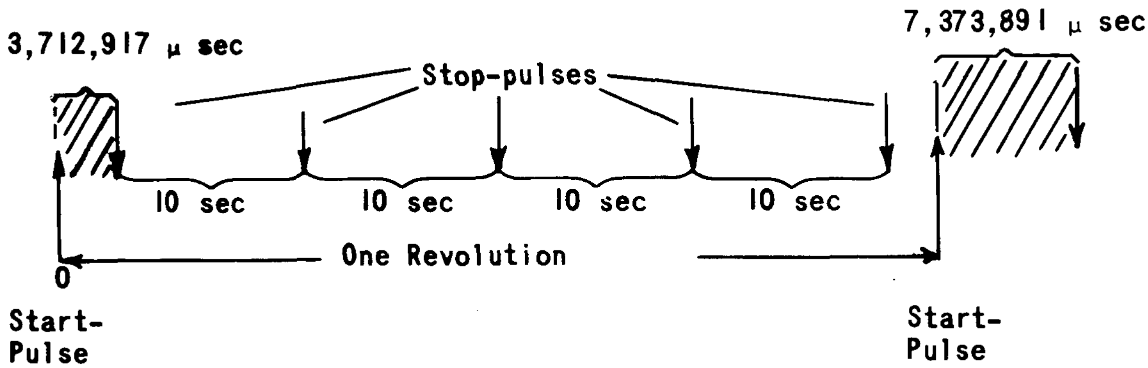
BLOCK DIAGRAM OF EQUIPMENT FOR MEASURING THE TIME PER REVOLUTION

After being amplified and shaped by the Schmitt Trigger (voltage level detector) the pulse from the photodiode starts the precision timer. Each ten seconds a pulse from the precision timer's reference is fed back into the precision timer and stops it. This stop pulse may occur any time from one microsecond to 9, 999, 999 microseconds after the start pulse. This precision time interval is recorded on the printer. The ten second pulse is recorded on the coarse timer. The time for each revolution is determined from the printed prevision readings and the number of ten second pulses that occur between start pulses.

An illustration of the timing of one revolution is as follows: Consider the first start pulse occurring at 0 time reference. The precision timer will print out a number such as 3, 712, 917 microseconds and five 10 second pulses may be counted. The second start pulse then occurs and the precision counter prints out 7, 373, 891 microseconds. This may be shown on a time axis as in Figure 3. The time of this particular revolution is then 46. 3990 seconds.

The accuracy of the method can be discussed as follows. The focal length of the mirror is 47. 5 inches, which places the apertures 95 inches from the mirror. The mirror's surface is 7. 5 inches from the axis of the rotating table, making the radial distance from the axis to the apertures 102. 5 inches. The diameter of the apertures and, therefore, the arc distance from the light going from completely off to full on is . 01 inches. The ratio of the arc distance to the radius is 10^{-4} radians. The light beam moves at twice the angular speed of the table making the resolution $.5 \times 10^{-4}$ radians.

The output from the photodiode will be shaped somewhat like a gaussian normal error curve with a peak amplitude of about 3 volts. The wave form is fed to a Schmitt Trigger whose output switches from one voltage level to another, a variation of 8 volts, as the input swings through a variation of only a few millivolts. Using this principle it seems reasonable to expect an improvement in the resolution by an order of 5. This order of 5 improvement over what the aperture provides gives a total resolution of 1×10^{-5} radians.



$$
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2{,}626{,}109 & = \quad 10{,}000{,}000 - 7{,}373{,}891 \\
\hline
46{,}339{,}026 &
\end{array}
$$

FIGURE 3

ILLUSTRATION OF TIMING PROCEDURE

To maintain 10^{-5} radian resolution, however, the position of the aperture must remain fixed to within 0.00025 inches! This allows 0.00025 inch variations in the triggering point on the waveform from the photodiode.

6. Suspension - Model B

This is the torsion fiber suspension. A prototype model using non-precision components for the large and small mass systems has been in operation for some time. Preliminary results which clearly demonstrate the feasibility of the basic experimental arrangement are presented in section C-1. The operating experience gained with this prototype has indicated several changes and modifications to the apparatus which are being incorporated in the design of a second prototype based on a model-A suspension. This is described in Section C-2.

7. Suspension - Model A

This is the model using the free electromagnetic suspension. The suspension system has been constructed and is currently nearing the completion of bench testing. It will then be inserted in prototype assembly No. 2 described in Section C-2.

8. Controlled Environment Facility

The prototype apparatus has been operating in a dust-free facility. However, the problem of temperature control has still not been resolved. It is expected that serious study on this problem will begin about May 1st, and that the necessary temperature control will be available by the end of the summer.

C. Task III - Assembly of Apparatus and Development of Techniques

1. Prototype Apparatus No. 1

In order to gain experience with the performance of the various units of the apparatus; a prototype system has been constructed. This uses a Model B or torsion fiber support and was described in some detail in the previous report (4).

The control loop on this prototype has now been closed and the apparatus has been operated extensively for the purpose of refining the control system. The results of a typical test are shown in Figure 4.

When using a torsion fiber suspension if the fiber is not at its null point, the resultant fibre torque will generate an acceleration of the system which is not related to the acceleration due to the gravitational torque. Since it is virtually impossible to start the system when the fibre is exactly at its null position, this background acceleration (or deceleration as the case may be) is usually present during these tests. In Figure 4 the measured angular velocity of the table is plotted against time (or revolutions) for an interval in which the table went through 15 revolutions.

The plot is clearly divided into three sections. At the beginning there were no large masses on the rotating table and the deceleration was caused by the restoring torque in the torsion fibre or other extraneous perturbations. At point A, two large masses were placed into position on the rotating table in an orientation so as to cause the gravitational torque to add to the deceleration (position A-A on inset). The resulting increased deceleration is apparent. At point B, the two large masses were removed and quickly placed in position B-B; where the gravitational torque now opposed and actually overpowered the residual fibre torque. As noted, the rotating table immediately responded by accelerating.

The large masses used in these experiments were very crude, of different materials, and irregular in shape, and so no quantitative significance can be attached to the results. However, qualitatively the experiments clearly demonstrate the feasibility of the technique for measuring gravitational attraction.

2. Prototype Apparatus No. 2

A second prototype model is now under construction. It has been designed specifically for a magnetic suspension of the small mass system (Model A) and it incorporates the modifications suggested by the experience obtained thus far. The new design is shown in some detail in Figure 5 in which an elevation view shows some of the important components in section, while a plan view is also sketched

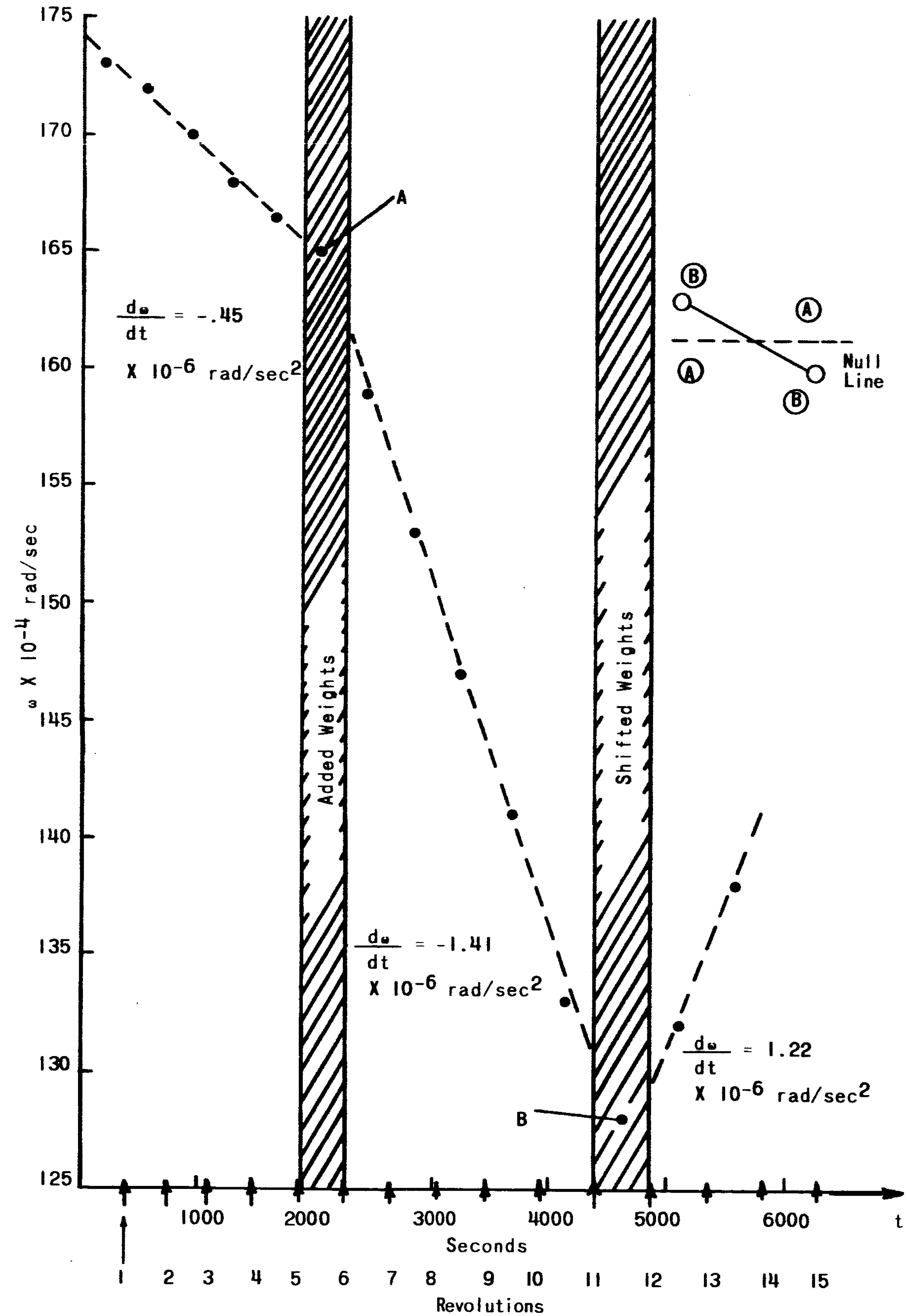

FIGURE 4

TYPICAL EXPERIMENT

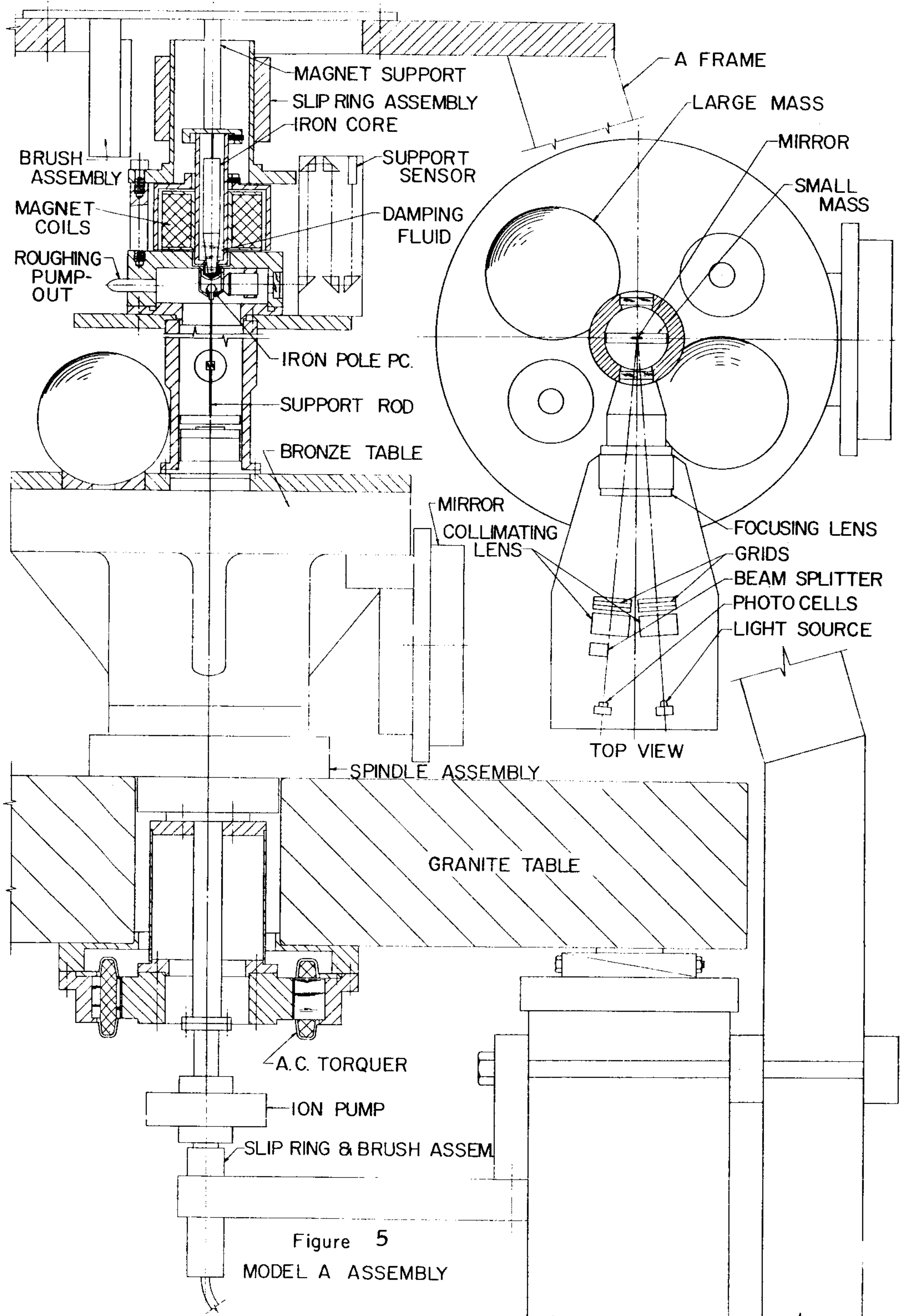

Figure 5
MODEL A ASSEMBLY

to provide orientation with the customary schematic of the experiment.

Among some of the principle changes from the first prototype, a photograph of which appeared in the previous report, are the following. The small mass system will now be a cylinder and a magnetic suspension will be used. The cylinder will be attached to a rigid support rod which contains a soft iron sphere on its upper end. This sphere will be attracted by the electromagnet and freely suspended by the usual, well-developed control techniques. Horizontal stability is provided by the iron magnet core suspended in a container of viscous fluid. The mirror on the support rod is part of the optical system which senses the rotation of the small mass and provides the "error" signal for controlling the rotating table. The arrangement shown is what has been designated as Model A-2 in which the magnet coil is attached rigidly to an A-frame support and is hence fixed in the laboratory system. It can be readily converted to a Model A-1 system by simply removing the magnet support column and the A-frame. This will allow the magnet to rest on the top of the vacuum housing for the support chamber and thus be fixed in the rotating system. As has been discussed in other reports, it is expected that there will be certain advantages and disadvantages to each arrangement.

Experience with the vacuum operation of the system has indicated it is not adequate to pump the support chamber through an external connection prior to a measurement and then operate on a static vacuum for extended periods of time. Consequently continuous pumping has been provided by attaching an ion pump to the rotating system which will pump through the hollow spindle.

The larger motor with radiation fins is now used and several modifications and improvements have been made in the slip ring assemblies.

3. <u>Metrology</u>

The problem of developing the measurement techniques required to determine to the desired precision all of the lengths, and distances involved in the basic apparatus remains a major problem.

Considerable thought and study, both on our part and in consultation with others, experts and experienced in the field, have been devoted to the problem. It seems as though the techniques to meet our requirements are in existence and the major undertaking is to adapt them successfully to make the measurements which must be made in situ.

A member of our staff attended a recent week long seminar on length measurements given by the National Bureau of Standards.

SECTION III
STAFF

The principal investigators gratefully acknowledge the assistance of the following staff members.

Drs. John Boring and Robert Humphris, members of the senior research staff of the Department of Aerospace Engineering and Engineering Physics, who have assumed responsibilities for various phases of the development.

Mr. James Dickerson, Design Engineer, who has developed the detailed specifications for many of the components procured commercially, and who has been responsible for the layout, construction and assembly of the two prototypes.

Messers. William Towler and Gerald Fisher, Electronic Engineers, who have developed the servo control for the rotary table, the magnetic suspension system, and the technique for measuring the period of rotation of the table.

Mrs. Jo Anne Kramer, a graduate student in Aerospace Engineering, who is performing the detailed calculations on many of the problems.

Finally much gratitude is due to Drs. A. V. McNish, T. R. Young and members of their staffs at the National Bureau of Standards, and to Messers. Roger Hibbs and Ed Bailey at the Y-12 Plant, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee, for their time and interest in the project which has resulted in many helpful suggestions and deeds.

It should be noted that Mr. J. P. Senter, who will shortly receive his doctorate in physics at the University of Virginia will join the project staff in June as a full time post-doctoral assistant. Mr. Senter will coordinate the many facets required to make the final transformation from prototype studies to the precision experiments.

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1. See e.g.

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